Pensieve Header: A 2D B-Picture.

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SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\w-Computations"]
```

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C:\drorbn\AcademicPensieve\Projects\w-Computations
```

```
ar[i_, j_] := t[i] h[j]
```

```
HMultiply[x_, y_, z_][mix_] := Module[
  {cox, coy, jx, jy, jxy},
  cox = D[mix, h[x]] /. t[s_] :> c[s];
  coy = D[mix, h[y]] /. t[s_] :> c[s];
  jx = If[cox === 0, 1, (-1 + e^cox)/cox];
  jy = If[coy === 0, 1, (-1 + e^coy)/coy];
  jxy = If[cox + coy === 0, 1, (-1 + e^(cox+coy))/(cox + coy)];
  (mix /. {h[x] -> 0, h[y] -> 0}) + jx h[z] D[mix, h[x]]/jxy + e^cox jy h[z] D[mix, h[y]]/jxy
]
```

```
BDisplay[expr_] := Collect[expr, _h, Collect[#, _t, FullSimplify] &]
```

```
HMultiply[3, 4, 5][t[1] h[3] + t[2] h[4]] // BDisplay
```

$$h[5]\left(\frac{\left(-1+e^{c[1]}\right)(c[1]+c[2])\,t[1]}{\left(-1+e^{c[1]+c[2]}\right)c[1]}+\frac{e^{c[1]}\left(-1+e^{c[2]}\right)(c[1]+c[2])\,t[2]}{\left(-1+e^{c[1]+c[2]}\right)c[2]}\right)$$

```
a = t[1] h[4] + t[2] h[5] + t[3] h[6]
```

```
h[4] t[1] + h[5] t[2] + h[6] t[3]
```

```
HMultiply[4, 5, 7][a] // BDisplay
```

$$h[7]\left(\frac{\left(-1+e^{c[1]}\right)(c[1]+c[2])\,t[1]}{\left(-1+e^{c[1]+c[2]}\right)c[1]}+\frac{e^{c[1]}\left(-1+e^{c[2]}\right)(c[1]+c[2])\,t[2]}{\left(-1+e^{c[1]+c[2]}\right)c[2]}\right)+h[6]\,t[3]$$

```
Expand2[expr_] := Series[expr /. {c[i_] :> z c[i], t[i_] :> z t[i]}, {z, 0, 2}]
```

```
HMultiply[4, 5, 7][a] // Expand2
```

$$(h[7]\,t[1]+h[7]\,t[2]+h[6]\,t[3])\,z+\frac{1}{2}\left(-c[2]\,h[7]\,t[1]+c[1]\,h[7]\,t[2]\right)z^2+O[z]^3$$

```
HMultiply[7, 6, 8][HMultiply[4, 5, 7][a]] // BDisplay
```

$$h[8]\left(\frac{\left(-1+e^{c[1]}\right)(c[1]+c[2]+c[3])\,t[1]}{\left(-1+e^{c[1]+c[2]+c[3]}\right)c[1]}+\right.$$
$$\left.\frac{e^{c[1]}\left(-1+e^{c[2]}\right)(c[1]+c[2]+c[3])\,t[2]}{\left(-1+e^{c[1]+c[2]+c[3]}\right)c[2]}+\frac{e^{c[1]+c[2]}\left(-1+e^{c[3]}\right)(c[1]+c[2]+c[3])\,t[3]}{\left(-1+e^{c[1]+c[2]+c[3]}\right)c[3]}\right)$$

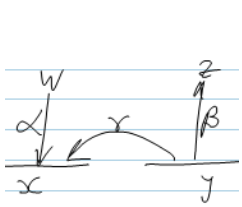
```
HMultiply[4, 7, 8][HMultiply[5, 6, 7][a]] // BDisplay
```

$$h[8]\left(\frac{\left(-1+e^{c[1]}\right)\left(c[1]+c[2]+c[3]\right)t[1]}{\left(-1+e^{c[1]+c[2]+c[3]}\right)c[1]}+\frac{e^{c[1]}\left(-1+e^{c[2]}\right)\left(c[1]+c[2]+c[3]\right)t[2]}{\left(-1+e^{c[1]+c[2]+c[3]}\right)c[2]}+\frac{e^{c[1]+c[2]}\left(-1+e^{c[3]}\right)\left(c[1]+c[2]+c[3]\right)t[3]}{\left(-1+e^{c[1]+c[2]+c[3]}\right)c[3]}\right)$$

```
(HMultiply[7, 6, 8][HMultiply[4, 5, 7][a]] -
   HMultiply[4, 7, 8][HMultiply[5, 6, 7][a]]) // BDisplay
```

0

From B – Side Operations, II :



$$E_y C_y^x(\mu) = \beta u_1 + \gamma u_2$$

$$= \beta^z \frac{\left(e^{\alpha^w c_w+\gamma c_y}-1\right)\alpha^w a_{wz} c_y + a_{yz}\left(\alpha^w c_w+\gamma c_y\right)}{e^{\alpha^w c_w+\gamma c_y}\,\alpha^w c_w+\gamma c_y} \;/.\; \begin{Bmatrix}\alpha\to-\alpha\\ \gamma\to-\gamma\end{Bmatrix}$$

$$+\gamma c_y \frac{e^{-\alpha^w c_w-\gamma c_y}-1}{-\alpha^w c_w-\gamma c_y}\, W+\gamma a_{yx}$$

```
THConjugate[y_, x_][mix_] := Module[
   {cox, coy, γ, δ, rest},
   cox = D[mix, h[x]] /. t[y] → 0;
   coy = D[mix, t[y]] /. h[x] → 0;
   γ = Coefficient[mix, t[y] h[x]];
   δ = -γ c[y] - (cox /. t → c);
   rest = mix /. t[y] → 0;
   coy (E^δ - 1) cox c[y] - t[y] δ / (E^δ (cox /. t → c) + γ c[y]) + γ c[y] (1 - E^-δ) / δ W + γ t[y] h[x] + rest
  ];
HTConjugate[x_, y_][mix_] :=
 ((mix /. h[x] → -h[x]) // THConjugate[y, x]) /. h[x] → -h[x]
THConjugate[y, x][t[y] h[1] + t[1] h[x]] // BDisplay
```

$$h[x]\,t[1]+h[1]\left(-\frac{\left(-1+e^{c[1]}\right)c[y]\,t[1]}{c[1]}+e^{c[1]}\,t[y]\right)$$

```
a1 = ar[1, 3] + ar[4, 2]
```

$$h[3]\,t[1]+h[2]\,t[4]$$

```
a2 = a1 // HTConjugate[3, 4] // HTConjugate[2, 4] // HMultiply[3, 2, 2] // BDisplay
```

$$-e^{-c[1]-c[4]}\left(-1+e^{c[4]}\right)W+h[2]\left(\frac{e^{c[4]}\left(-1+e^{c[1]}\right)\left(c[1]+c[4]\right)t[1]}{\left(-1+e^{c[1]+c[4]}\right)c[1]}+\frac{\left(-1+e^{c[4]}\right)\left(c[1]+c[4]\right)t[4]}{\left(-1+e^{c[1]+c[4]}\right)c[4]}\right)$$

a3 = a1 // HMultiply[2, 3, 2] // HTConjugate[2, 4] // BDisplay

$$-e^{-c[1]-c[4]}\left(-1+e^{c[4]}\right)W+h[2]\left(\frac{e^{c[4]}\left(-1+e^{c[1]}\right)(c[1]+c[4])\,t[1]}{\left(-1+e^{c[1]+c[4]}\right)c[1]}+\frac{\left(-1+e^{c[4]}\right)(c[1]+c[4])\,t[4]}{\left(-1+e^{c[1]+c[4]}\right)c[4]}\right)$$

FullSimplify[a2 == a3]

True

ComposeList[{HTConjugate[3, 4], HTConjugate[2, 4], HMultiply[2, 3, 2]}, a1] // BDisplay // MatrixForm

$$\begin{pmatrix} h[3]\,t[1]+h[2]\,t[4] \\ h[3]\,t[1]+h[2]\left(\frac{c[4]\,(1-\text{Cosh}[c[1]]+\text{Sinh}[c[1]])\,t[1]}{c[1]}+e^{-c[1]}\,t[4]\right) \\ -e^{-c[1]-c[4]}\left(-1+e^{c[4]}\right)W+h[3]\,t[1]+h[2]\left(\frac{c[4]\,(1-\text{Cosh}[c[1]]+\text{Sinh}[c[1]])\,t[1]}{c[1]}+e^{-c[1]}\,t[4]\right) \\ -e^{-c[1]-c[4]}\left(-1+e^{c[4]}\right)W+h[2]\left(\frac{e^{-c[1]}\left(-1+e^{c[1]}\right)\left(-1+e^{c[4]+c[1]+c[4]}\right)(c[1]+c[4])\,t[1]}{\left(-1+e^{c[1]+c[4]}\right)c[1]}+\frac{e^{-c[1]}\left(-1+e^{c[4]}\right)(c[1]+c[4])\,t}{\left(-1+e^{c[1]+c[4]}\right)c[4]}\right) \end{pmatrix}$$

ComposeList[{HMultiply[2, 3, 2], HTConjugate[2, 4]}, a1] // BDisplay // MatrixForm

$$\begin{pmatrix} h[3]\,t[1]+h[2]\,t[4] \\ h[2]\left(\frac{e^{c[4]}\left(-1+e^{c[1]}\right)(c[1]+c[4])\,t[1]}{\left(-1+e^{c[1]+c[4]}\right)c[1]}+\frac{\left(-1+e^{c[4]}\right)(c[1]+c[4])\,t[4]}{\left(-1+e^{c[1]+c[4]}\right)c[4]}\right) \\ -e^{-c[1]-c[4]}\left(-1+e^{c[4]}\right)W+h[2]\left(\frac{e^{c[4]}\left(-1+e^{c[1]}\right)(c[1]+c[4])\,t[1]}{\left(-1+e^{c[1]+c[4]}\right)c[1]}+\frac{\left(-1+e^{c[4]}\right)(c[1]+c[4])\,t[4]}{\left(-1+e^{c[1]+c[4]}\right)c[4]}\right) \end{pmatrix}$$

b1 = ar[1, 2] + ar[4, 3]

$$h[2]\,t[1]+h[3]\,t[4]$$

b2 = b1 // HTConjugate[3, 4] // HTConjugate[2, 4] // HMultiply[3, 2, 2] // BDisplay

$$\left(-1+e^{-c[4]}\right)W+h[2]\left(\frac{e^{-c[1]}\left(-1+e^{c[1]}\right)\left(-1+e^{c[4]+c[1]+c[4]}\right)(c[1]+c[4])\,t[1]}{\left(-1+e^{c[1]+c[4]}\right)c[1]}+\frac{e^{-c[1]}\left(-1+e^{c[4]}\right)(c[1]+c[4])\,t[4]}{\left(-1+e^{c[1]+c[4]}\right)c[4]}\right)$$

b3 = b1 // HMultiply[2, 3, 2] // HTConjugate[2, 4] // BDisplay

$$\left(-1+e^{-c[4]}\right)W+h[2]\left(\frac{\left(-1+e^{c[1]}\right)(c[1]+c[4])\,t[1]}{\left(-1+e^{c[1]+c[4]}\right)c[1]}+\frac{e^{c[1]}\left(-1+e^{c[4]}\right)(c[1]+c[4])\,t[4]}{\left(-1+e^{c[1]+c[4]}\right)c[4]}\right)$$

FullSimplify[b2 == b3]

$$\frac{e^{-c[1]}\left(-1+e^{2c[1]}\right)\left(-1+e^{c[4]}\right)(c[1]+c[4])\,h[2]\,(-c[4]\,t[1]+c[1]\,t[4])}{\left(-1+e^{c[1]+c[4]}\right)c[1]\,c[4]}==0$$

c1 = ar[1, 2]

$$h[2]\,t[1]$$

c1 // THConjugate[1, 2]

$$-\left(1-e^{c[1]}\right)W+h[2]\,t[1]$$